Performance Comparison of Evolutionary Algorithms Applied to Hybrid Rocket Problem

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***Abstract*—Optimizer is an essence in design-informatics to efficiently obtain nondominated solutions. In the present study, the best optimizer is decided through the competition for the mathematical standard test functions and is also applied to a conceptual design problem of single-stage simple hybrid rocket as the real-world problem. Consequently, a hybrid method between differential evolution and genetic algorithm has good exploration performance. Moreover, the principal component analysis blended crossover and the confidence interval based crossover have good capability on the hybrid method between differential evolution and genetic algorithm.**

I. Introduction

The results produced by multiobjective (MO) optimization are not an individual optimal solution but rather an entire set of optimal solutions. This set generated by an MO optimization can be considered a hypothetical design database. Then, data mining techniques can be applied to this hypothetical database to acquire not only useful design knowledge but also structuring and visualizing of design space. This approach was suggested as design-informatics[1]. The design problem is firstly defined as objective functions, constraints, and design space. And then, optimization is implemented to obtain nondominated solutions for database construction. The purpose of this approach is the conception support for designers in order to materialize innovation. This methodology is constructed by the three essences as problem definition, optimization, and information mining. In this study, optimizer for efficient exploration in design space is focused because the quality of hypothetical design database depends on that. The objective of this study is the evaluation of several evolutionary algorithms and their hybrid methods.

In the present study, the practical engineering application with large evaluation time is assumed. Therefore, the evolutionary optimizer which efficiently explores in a small number of generations is needed. Differential evolution has recently better performance than genetic algorithm in MO optimization[2]. Then, the performance of a genetic algorithm (GA)[3], a differential evolution (DE)[4], a particle swarm optimization (PSO)[5](GA and DE have advantage for global search while PSO is advantage for local search), and their hybrid methods is validated to employ practical engineering applications. Hence, they evaluate under the condition of a small number of population and generation. The qualitative performance is evaluated for the mathematical standard test functions for which the influence of noise are considered. As a next step, the capability of crossover is investigated for the same test functions. Their optimization methods are also applied to single-stage simple hybrid rocket design in order to refine the problem definition.

II. Optimizers

A. Hybrid Algorithm

Three optimizers, as GA, DE, and PSO, are coupled. First, multiple solutions are generated randomly as an initial population. Then, objective function values are evaluated for each solution. After the evaluation, the populations is equally divided into sub-populations for the operations in each optimizer(as this sub-population size can be decided in every generation, pure GA can be single performed when the sub-populations of DE and PSO are zero for example). New solutions generated by each operation are combined in the next generation. Nondominated solutions in the combined population are archived. It is notable that only the archive data are shared among the each optimizer, the respective optimizers are independently carried out in the hybrid algorithm. Therefore, the total number of seven optimizers were evaluated as pure GA, pure DE, pure PSO, hybrid GA/PSO, hybrid DE/PSO, hybrid GA/DE, and hybrid GA/DE/PSO. It is notable that the range adaptation was performed at every 20 generations.

B. Configuration of GA Operators

Fonseca's Pareto ranking[6] and crowding distance[7] were used as the fitness value of each solution. The crowding distance was defined as the sum of Euclidean distances between the solution and its two nearest neighbors. As crossover operators, the blended crossover(BLX)-α and the unimodal normal distribution crossover(UNDX) were used, which equally divided sub-population.

III. Evaluation Using Test Functions

A. Performance Metrics

Several performance measurement manners for evaluating the efficiency of MOEAs were suggested[8]. In this study, the following three metrics were used.

1) Convergence Metric: The first metric is *Convergence metric* Υ[9]. It measures the distance between the obtained non-dominated front Q and the set P^* of Pareto-optimum solutions as follows:

$$\Upsilon = \frac{1}{|Q|}\sum_{i \in Q} d_i, \tag{1}$$

where d_i denotes the Euclidean distance in the objective-function space between the solution $i \in Q$ and the nearest member of P^*. The value near zero means better performance.

2) Cover Rate: The second metric is *Cover rate* R_c[10]. R_c evaluates the width and closeness of non-dominated solutions compared with Pareto-optimum front. The design space closed by the objective values from minimum to maximum is taken discretization. This metric describes the degree that non-dominated solutions cover discrete region. In this study, two-/three-dimensional test functions are evaluated. The objective-function space is separated by squares and cubes. The cover rate R_c is the following equation:

$$R_c = \frac{N_{\mathrm{NDS}}}{N_{\mathrm{Pareto}}}, \tag{2}$$

where N_{NDS} denotes the number of the cubes included in the derived non-dominated solutions. N_{Pareto} denotes the number of the cubes intersected by the Pareto front. The maximum value of R_c gives one and the minimum value of R_c gives zero, and then the value near one means better performance.

3) Hypervolume: The hypervolume indicator (or S metric) is described as the Lebesgue measure Λ of the union of hypercubes a_i defined by a non-dominated point m_i and a reference point x_{ref}[11]:

$$\begin{aligned} S(M) &\overset{\mathrm{def}}{\equiv} \Lambda\left(\left\{\bigcup_i a_i | m_i \in M\right\}\right) \\ &= \Lambda\left(\bigcup_{m \in M} \{x | m \prec x \prec x_{\mathrm{ref}}\}\right) \end{aligned} \tag{3}$$

B. Test Function

Three standard test function problems are employed in order to evaluate the performance of optimizers under the consideration of noise described by using normal distribution with random number. The first function is DTLZ3[12] without noise using three objective functions and 10 design variables. The second function is ZDT1[13] with/without noise, which is a simple two-dimensional problem with 10 design variables. The final function is TNK[14] with noise as a constraint two-dimensional test function. When an optimizer is applied to practical problems, experimental and computational values are employed as those of objective functions. Experiment includes error due to the flow quality in wind tunnel. Computation (computational fluid dynamics *etc.*) similarly has error due to mesh and various modeling *etc*. That is, as noise is occurred for the evaluated value under an identical condition, the consideration of noise is important to investigate the performance of optimizer applicable to practical engineering problem.

1) DTLZ3: This is a generic sphere problem. The Pareto-optimal surface always occurs for the minimum of $g(\boldsymbol{x})$ function. The number of design variables and objective functions set in this paper were 10 and three for DTLZ3.

$$\begin{aligned} &\text{Minimize:} && f_1(\boldsymbol{x}) = \cos\left(\frac{\pi}{2}x_1\right)\cos\left(\frac{\pi}{2}x_2\right)(1+g(\boldsymbol{x})) \\ &\text{Minimize:} && f_2(\boldsymbol{x}) = \cos\left(\frac{\pi}{2}x_1\right)\sin\left(\frac{\pi}{2}x_2\right)(1+g(\boldsymbol{x})) \\ &\text{Minimize:} && f_3(\boldsymbol{x}) = \sin\left(\frac{\pi}{2}x_2\right)(1+g(\boldsymbol{x})) \\ &\text{subject to:} && g(\boldsymbol{x}) = 100\times \\ & \left[k + \sum_{k=3}^{K}\left\{(x_k-0.5)^2 - \cos\left(20\pi\left(x_k-0.5\right)\right)\right\}\right] \geq 0, \\ & && 0 \leq x_k \leq 1, \quad k = 1,2,\cdots,K, \quad K = 10. \end{aligned} \tag{4}$$

The Pareto-optimum solution corresponds to $x_i = 0.5$ (for all $x_i \in \boldsymbol{x}$) and the objective function values lie inside the first octant of the unit sphere $\sum_{m=1}^{3} f_m = 1$ in a three-objective plot. All local Pareto-optimal fronts are parallel to the global Pareto-optimal front and an MOEA can get stuck at any of these local Pareto-optimal fronts, before converging to the global Pareto-optimal front.

2) ZDT1: As a test problem with noise, the following two-dimensional test function was considered:

$$\begin{aligned} &\text{Minimize:} && f_1(\boldsymbol{x}) = x_1 \\ &\text{Minimize:} && f_2(\boldsymbol{x}) = g(\boldsymbol{x})\left(1 - \sqrt{\frac{f_1(\boldsymbol{x})}{g(\boldsymbol{x})}}\right) \\ &\text{subject to:} && g(\boldsymbol{x}) = 1 + 9\cdot\frac{1}{K-1}\sum_{k=2}^{K} x_k, \\ & && 0 \leq x_k \leq 1, \quad k = 1,2,\cdots,K, \quad K = 30. \end{aligned} \tag{5}$$

The Pareto-optimum front is formed with $g(\boldsymbol{x}) = 1$. As noise is appended to this test function, the performance for noise occurred in practical problems is confirmed.

3) TNK: As a test problem with noise, the following two-dimensional test function was considered:

$$\begin{aligned} &\text{Minimize:} && f_1(\boldsymbol{x}) = x_1 \\ &\text{Minimize:} && f_2(\boldsymbol{x}) = x_2 \\ &\text{subject to:} && c_1(\boldsymbol{x}) = x_1^2 + x_2^2 \\ & && \quad -1 - 0.1\cos\left(16\arctan\frac{x_2}{x_1}\right) \geq 0 \\ & && c_2(\boldsymbol{x}) = \left(x_1 - \frac{1}{2}\right)^2 + \left(x_2 - \frac{1}{2}\right)^2 \leq \frac{1}{2} \\ & && 0 < x_i \leq \pi, \quad i = 1,2. \end{aligned} \tag{6}$$

This is a two real-valued variable constrained test problem. Since the function is simple and the objective-function space corresponds to the design-variable space, the Pareto front is determined by the constraints. As this function is a minimization problem, the discontinuous region which is not dominated by the other region in the curve described by $c_1(\boldsymbol{x}) = 0$. The ratio which the feasible region accounts is approximately 5% of the whole region. The Pareto front of this test function is non-convex surface. Therefore, this test function with noise reveals the performance for intricate practical problems.

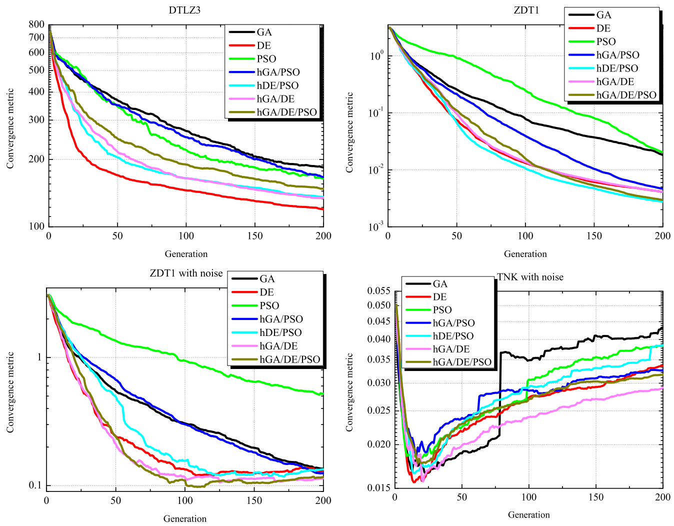

Fig. 1. Histories of convergence metric for each optimizer.

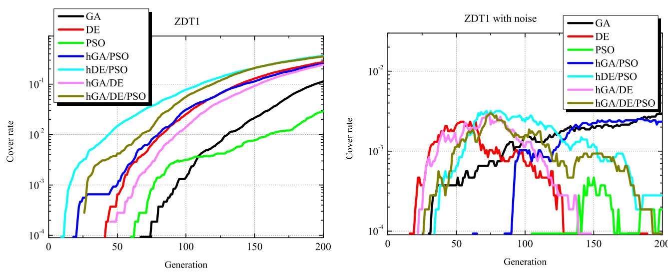

Fig. 2. Histories of cover rate for each optimizer. As all data for DTLZ3 and TNK are zero, their histories were omitted.

C. Results

The population size and the maximum number of generations were respectively set to be 18 and 200. As the purpose of performance evaluation for several optimizations is to be applied to large-scale and real-world engineering design problem(for example, it takes one week for one-generation evaluation), comparatively small values were used. It is notable that the average values of 20 runs with different initial populations generated randomly were employed for evaluation.

The histories of convergence metric shown in Fig. 1 reveal that pure DE and the hybrid methods including DE have good performance. DE sustains damageless from noise. GA does not have much influence from noise. Although pure PSO has poor performance regarding noise, the hybridization with DE improve it. The hybridization between GA and DE gives the potentiating effect for the performance. Although the hybridization between PSO and the others also gives similar effects, the frailty of pure PSO for noise is bottleneck.

The histories of cover rate shown in Fig. 2 also reveal that pure DE and the hybrid methods including DE have robustness for noise. GA does not have good performance. DE has adamant performance for noise, and also the hybrid methods including DE maintain similar robustness. Although pure PSO is frail for noise, the hybridization including PSO has compatibility.

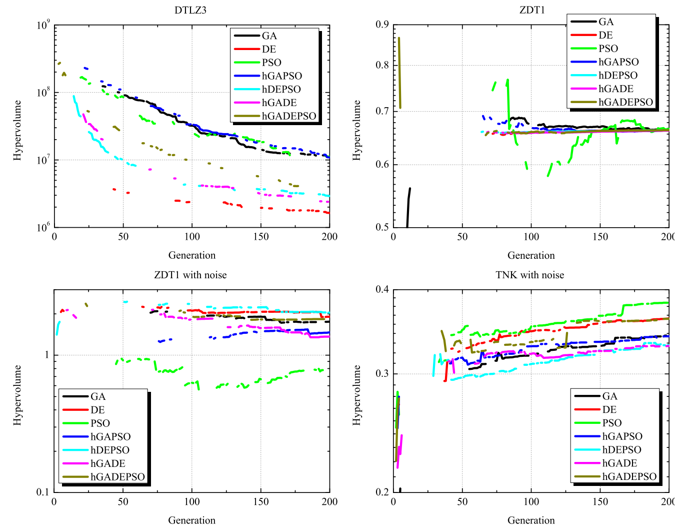

Fig. 3. Histories of hypervolume for each optimizer.

The histories of hypervolume shown in Fig. 3 also reveal that pure DE does not have good performance in the case without noise, but has robustness in the case with noise. DE has stable performance. Pure GA has good performance in the case without noise, however, it is not good and problem dependency in the case with noise. Pure PSO is unstable in spite of noise and its performance depends on the test problems. In the case without noise, pure DE and the hybridization using DE are better. The hybridization including DE is also good for the problems with noise due to the restoration of pure DE performance. Note that there is no meaningful difference regarding the results for TNK.

As a result, a hybrid method between GA and DE will be selected to apply to a large-scale engineering design problem because pure DE is robust and stable for noise and pure GA is expected to have latent performance due to complex operator compared with DE. PSO which does not have the strength for noise and the hybrid method with PSO should not be selected because practical engineering design problem certainly includes noise.

IV. Application to Hybrid Rocket Problem

The conceptual design for a single-stage simple hybrid rocket[15], which is composed of a thrust chamber, an oxidizer tank, a nozzle, and a payload, is considered in the present study. This problem is developing and modifying for the evaluation of optimizer applicable to large-scale and real-world design problem. Here, the seven optimizers evaluated by the standard test functions are applied to this hybrid rocket problem, and then the refinement points of it will be confirmed as well as the performance of each optimizer is compared.

A. Objective Functions

Two objective functions are defined in this study. One is the maximization of maximum altitude $H_{\max}$ [km] and the other is the minimization of gross vehicle weight $M_{\rm tot}(0)$ [kg]. Note that three other objective functions can be added for this design problem.

B. Design Variables

Six design variables are used as initial mass flow of oxidizer $\dot{m}_{\rm oxi}(0)$ [kg/sec], fuel length $L_{\rm fuel}$ [m], initial radius of port $r_{\rm port}(0)$ [m], combustion time $t_{\rm burn}$ [sec], initial pressure in combustion chamber $P_{cc}(0)$ [MPa], and aperture ratio of nozzle ϵ [-].

C. Evaluation Methods

1) Trajectory Analysis: The following equation of motion described by using thrust $T(t)$ [N] and drag $D(t)$ [N] is computed.

$$M_{\rm tot}(t)\left\{a(t)-g\right\} = T(t) - D(t) \tag{7}$$

$T(t)$ is evaluated by using the following equation.

$$T(t) = \eta_T \left\{\lambda \dot{m}_{\rm prop}(t)\cdot u_e + (P_e - P_a)\cdot A_e\right\} \tag{8}$$

where, η_T is total thrust loss coefficient, λ is momentum loss coefficient at nozzle exit by friction, $\dot{m}_{\rm prop}(t)$ is mass flow of propellant, u_e is velocity at nozzle exit, P_e is pressure at nozzle exit, P_a is pressure of atmosphere at flight altitude, and A_e describes area of nozzle exit.

$$\begin{aligned} \dot{m}_{\rm prop}(t) &= -(\dot{m}_{\rm oxi}(t) + \dot{m}_{\rm fuel}(t)) \\ \dot{m}_{\rm fuel}(t) &= 2\pi r_{\rm port}(t) L_{\rm fuel}\rho_{\rm fuel}\ \bar{\dot{r}}_{\rm port}(t) \\ r_{\rm port}(t) &= r_{\rm port}(0) + \int \dot{r}_{\rm port}(t)dt \end{aligned} \tag{9}$$

Figure x shows the definition of shape and symbols. A combustion chamber has solid fuel with a single port to supply oxidizer. The regression rate to the radial direction of the fuel $\dot{r}_{\rm port}(t)$ [m/sec] generally governs the thrust power of hybrid rocket engine.

$$\begin{aligned} \dot{r}_{\rm port}(t) &= 8.26\times 10^{-5}\times G_{\rm oxi}^{0.55}(t) \\ &= 8.26\times 10^{-5}\times \left(\frac{\dot{m}_{\rm oxi}(t)}{\pi r_{\rm port}^2(t)}\right)^{0.55} \end{aligned} \tag{10}$$

where, $G_{\rm oxi}$ is oxidizer mass flux [kg/(m^2 sec)], $\dot{m}_{\rm oxi}(t)$ is oxidizer flow [kg/sec], and $r_{\rm port}(t)$ describes radius of port [m].

$D(t)$ is described by using pressure drag $D_p(t)$ and friction drag $D_f(t)$ and is respectively estimated by using the flight data of S-520 as the solid rocket in Japan Aerospace Exploration Agency.

$$\begin{aligned} D(t) &= D_p(t) + D_f(t) \\ D_p(t) &= \frac{1}{2}\rho V^2 S_{\rm ref} C_{D_p}^{(\rm S-520)} \\ D_f(t) &= \frac{1}{2}\rho V^2 S_{\rm tot} C_{D_f} \end{aligned} \tag{11}$$

where, $S_{\rm ref}$ is reference area and $S_{\rm tot}$ is total surface area.

$$\begin{aligned} C_{D_p}^{(\rm S-520)} &= C_D^{(\rm S-520)} - C_{D_f}^{(\rm S-520)}\cdot\frac{S_{\rm tot}^{(\rm S-520)}}{S_{\rm ref}^{(\rm S-520)}} \\ C_{D_f}^{(\rm S-520)} &= \frac{0.455}{(\log_{10} Re)^{2.58}}\cdot\frac{1}{(1+0.144M^2)^{0.655}} \\ Re &= \frac{VL_{\rm tot}^{\rm S-520}}{\nu} \\ M &= \frac{V}{\sqrt{\gamma RT}} \end{aligned} \tag{12}$$

where, $L_{\rm tot}^{\rm S-520} = 8.715$ [m], specific heat ratio $\gamma = 1.4$, and gas constant $R = 287$ [J/(kg·K)].

$$\begin{aligned} C_{D_f} &= \frac{0.455}{(\log_{10} Re)^{2.58}}\cdot\frac{1}{(1+0.144M^2)^{0.655}} \\ Re &= \frac{VL_{\rm tot}}{\nu} \end{aligned} \tag{13}$$

Kinematic viscosity coefficient ν and atmospheric temperature T are variables for altitude, referred by International Standard Atmosphere.

2) Structure Analysis: Initial gross weight is evaluated by the following equation.

$$\begin{aligned} M_{\rm tot}(0) &= \frac{M_{\rm prop}(0)}{0.65} + M_{\rm pay} \\ &= \frac{1}{0.65}(M_{\rm oxi} + M_{\rm fuel}) + M_{\rm pay} \\ M_{\rm oxi} &= \int_0^{t_{\rm burn}} \dot{m}_{\rm oxi}(t)dt \\ M_{\rm fuel} &= \int_0^{t_{\rm burn}} \dot{m}_{\rm fuel}(t)dt \end{aligned} \tag{14}$$

Constant value of 0.65 means that mass of propellant assumes 65% of gross weight. $M_{\rm pay}$ describes mass of payload.

D. Applied Results

Evolutions were performed until 100th generation by using 18 population per generation for all of the optimizers because the nondominated solutions obtained at 100th generation already converged shown in Fig. 4. Figure 4 indicates that the present design problem for hybrid rocket is simple because of the mildness between the objective function and the design variable. In fact, the nondominated solutions generates Pareto surface at a small number of generation. Therefore, severe design variable should be added and redefined the present design problem not only for the severeness of evaluation tool but also for the growth of conceptual design of hybrid rocket. The mass flow of oxidizer should be a design variable as one of idea. The influence of gust at flight should also considered as a perturbation.

Figure 5 shows the histories of the number of nondominated solutions and that of hypervolume. This figure shows that PSO has a large number of nondominated solutions, but it has low hypervolume indicator. On the other hand, the hybrid method between DE and GA does not have a large number

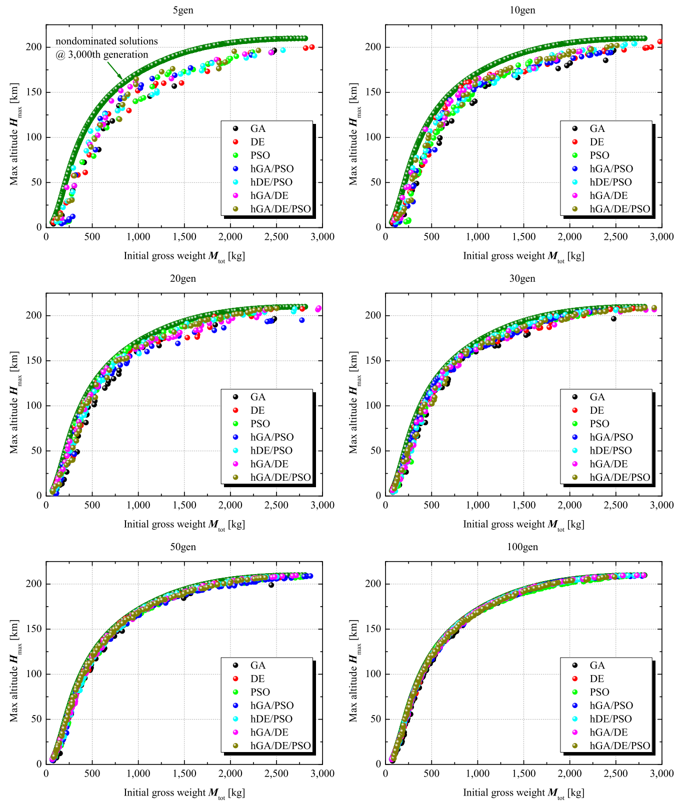

Fig. 4. Comparison of nondominated solutions at several generation for each optimizer.

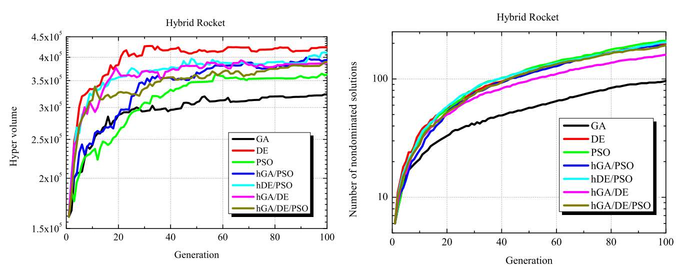

Fig. 5. History of hypervolume and number of nondominated solutions for each optimizer.

of nondominated solutions compared with PSO results, but it has high hypervolume indicator. That is, the hybrid method between DE and GA implements the global exploration in design space and maintains the diversity of nondominated solutions, whereas PSO carries out the local search and it has poor diversity. Therefore, the results indicate the high performance of the hybrid method between DE and GA.

V. Capability of Crossover on Hybrid Method between DE and GA

The capability of the eight crossovers on the hybrid method between DE and GA. GA with BLX-α and UNDX was used on the above problem for the comparison among pure and hybrid methods. As a result, pure DE has better performance than pure GA. Reference [16] shows the GA with the simplex crossover(SPX) reaches better performance for convergence

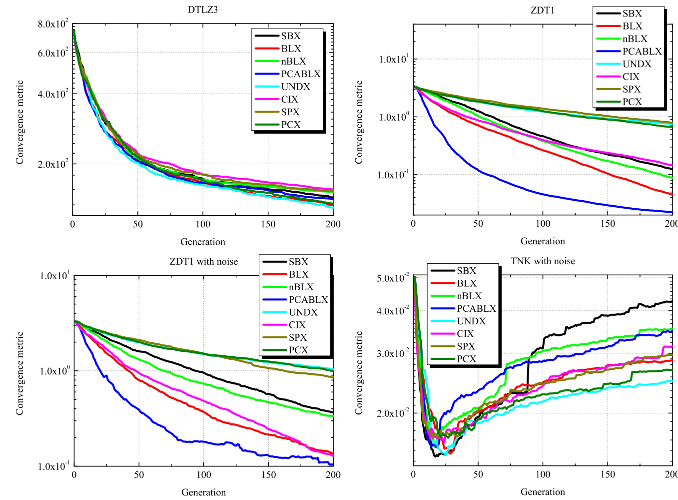

Fig. 6. Histories of convergence metric for each crossover.

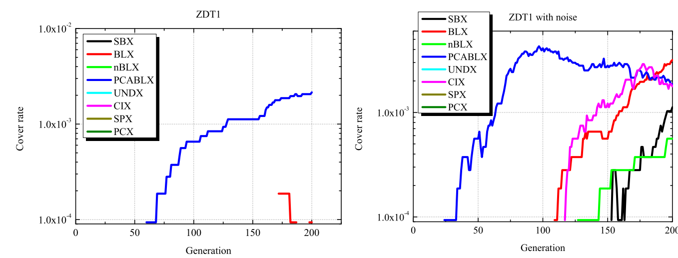

Fig. 7. History of cover rate for each crossover. As all data for DTLZ3 and TNK are zero, their histories were omitted.

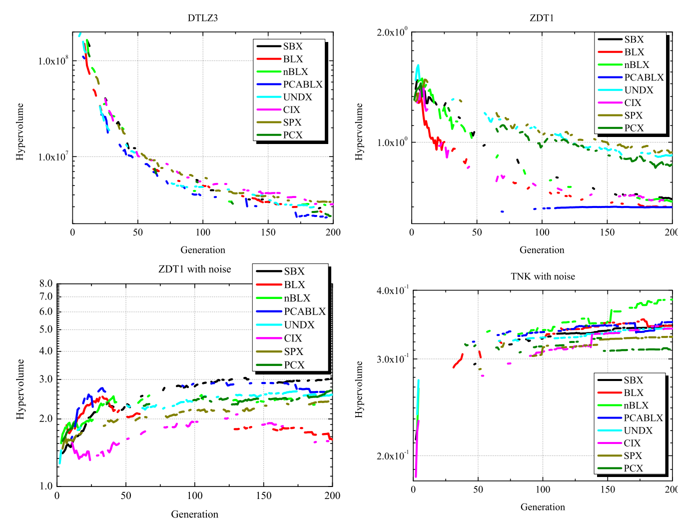

Fig. 8. History of hypervolume for each crossover.

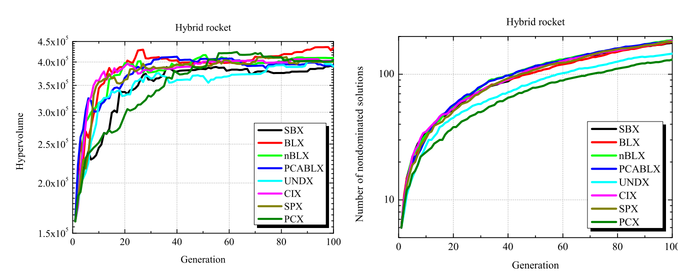

Fig. 9. Histories of hypervolume and number of nondominated solutions for each crossover on hybrid rocket problem.

metric. Therefore, the difference of the capability regarding crossover will be confirmed. The computational conditions were similar to the above problem to compare among pure and hybrid optimizers. The eight crossovers were employed as the simulated binary crossover(SBX), BLX-α, BLX-α with neighborhood selection method(nBLX), the principal component analysis BLX-α(PCABLX), UNDX, the confidence interval based crossover using L^2 norm(CIX), SPX, and the parent-centric crossover(PCX).

The histories of the convergence metric for each crossover shown in Fig. 6 indicate that PCABLX is better performance. PCABLX has the strength for noise. Although the convergence metric for TNK with noise shows that CIX is good performance at early generation, there is no significant difference among all crossovers. The histories of the cover rate shown in Fig. 7 indicate the better capability of PCABLX. The histories of the hypervolume shown in Fig. 8 also indicate that PCABLX is good performance for the problem without noise and also CIX is good capability for the problem with noise. Consequently, PCABLX and CIX are the good selection for an unknown problem. The history of hypervolume for the hybrid rocket problem shown in Fig. 9 shows PCABLX is good performance at early generation, and then CIX is good capability. When the result of hybrid rocket problem are observed based on the above those for the test functions, this problem has week noise. Therefore, this problem is not a simple, however, is insufficient as a real-world problem. The several refinements will be implemented in the next step.

VI. Conclusions

Pure three evolutionary-based optimizers as a differential evolution, a genetic algorithm, and a particle swarm optimization, and their hybrid methods have been compared among mathematical standard test functions with noise. As a result, a hybrid method between differential evolution and genetic algorithm was selected because of the high performance for convergence metric, cover rate, and hypervolume. The capability of crossover was also investigated by using eight as the simulated binary crossover, the blended crossover, the blended crossover with neighborhood selection method, the principal component analysis blended crossover, the unimodal normal distribution crossover, the confidence interval based crossover using L^2 norm, the simplex crossover, and the parent-centric crossover. As a result, the principal component analysis blended crossover and the confidence interval based crossover had good capability on the hybrid method between differential evolution and genetic algorithm. Moreover, a real-world hybrid rocket design problem under the single-stage simple condition is applied in order to refine the problem definition and also to evaluate the above optimizers and crossovers. Consequently, a hybrid method between differential evolution and genetic algorithm was also good exploration performance in the design space. The present design problem for hybrid rocket would be updated as more severe evaluation problem.

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References

[1] K. Chiba, Y. Makino, and T. Takatoya, “Design-informatics approach applicable to real-world problem,” in *Proceedings on IEEE Symposium on Computational Intelligence 2011 in Multicriteria Decison-Making*, 2011, pp. 167–174.

[2] T. Tušar and B. Filipič, “Differential evolution versus genetic algortihms in multiobjective optimization,” in *4th International Conference on Evolutionary Multi-Criterion Optimization, LNCS 4403*, Springer-Verlag Heidelberg, Matsushima, Japan, 2007, pp. 257–271.

[3] D. Sasaki and S. Obayashi, “Efficient search for trade-offs by adaptive range multi-objective genetic algorithms,” *Journal of Aerospace Computing, Information, and Communication*, vol. 2, no. 1, pp. 44–64, 2005.

[4] R. Storn and K. Price, “Differeintial evolution – a simple and efficient heuristic for global optimization over continuous space,” *Journal of Global Optimization*, vol. 11, no. 4, pp. 341–359, 1997.

[5] J. E. Alvarez-Benitez, R. M. Everson, and J. E. Fieldsend, “A mopso algorithm based exclusively on pareto dominance concepts,” in *The 3rd International Conference on Evolutionary Multi-Criterion Optimization, LNCS 3410*, Springer-Verlag Heidelberg, Guanajuato, Mexico, 2005, pp. 459–473.

[6] C. M. Fonseca and P. J. Fleming, “Genetic algorithms for multiobjective optimization: Formulation, discussion and generalization,” in *Proceedings of the Fifth International Conference on Genetic Algorithms*, 1993, pp. 416–423.

[7] K. Deb, S. Agrawal, A. Pratap, and T. Meyarivan, “A fast elitist non-dominated sorting genetic algorithm for multi-objective optimization,” in *Proceedings Parallel Problem Solving from Nature IV*, 2000, pp. 849–858.

[8] J. D. Knowles, L. Thiele, and E. Zitzler, “A tutorial on the performance assessment of stochastic multiobjective optimizers,” in *TIK-Report No. 214 Revised Version*, 2006, pp. 1–35.

[9] T. Robič and B. Filipič, “Demo: Differential evolution for multiobjective optimization,” in *The 3rd International Conference on Evolutionary Multi-Criterion Optimization, LNCS 3410*, Splinger-Verlag, Guanajuato, Mexico, 2005, pp. 520–533.

[10] T. Hiroyasu, M. Miki, and S. Watanabe, “Distributed genetic algorithms with a new sharing approach in multiobjective optimization problems,” in *Proceedings on IEEE Congress on Evolutionary Computation 1999*, 1999, pp. 69–76.

[11] E. Zitzler, D. Brockhoff, and L. Thiele, “The hypervolume indicator revisited: On the design of pareto-compliant indicators via weighted integration,” in *The 4th International Conference on Evolutionary Multi-Criterion Optimization, LNCS 4403*, Splinger-Verlag, Matsushima Japan, 2007, pp. 862–876.

[12] K. Deb, L. Thiele, M. Laumanns, and E. Zitzler, “Scalable test problems for evolutionary multi-objective optimization,” in *Evolutionary Computation Based Multi-Critera Optimization: Theoretical Advances and Applications*, Springer-Verlag, 2005, pp. 1–35.

[13] E. Zitzler, K. Deb, and L. Thiele, “Comparison of multiobjective evolutionary algorithms: Empirical results,” *Evolutionary Computation*, vol. 8, no. 2, pp. 173–195, 2000.

[14] M. Tanaka, H. Watanabe, Y. Furukawa, and T. Tanino, “Ga-based decision support system for multicriteria optimization,” in *Proceedings on IEEE International Conference on Systems, Man and Cybernetics*, NY, USA, 1995, pp. 1556–1561.

[15] Y. Kosugi, A. Oyama, K. Fujii, and M. Kanazaki, “Multidisciplinary and multi-objective desgin exploration methodology for conceptual design of a hybrid rocket,” 2011, aIAA Paper 2011-1634, 2011.

[16] A. Iorio and X. Li, “Rotationally invariant crossover operators in evolutionary multi-objective optimization,” in *6th International Conference on Simulated Evolution And Learning, LNCS 4247*, Springer-Verlag Heidelberg, Hefei, China, 2006, pp. 310–317.