Pure and Hybrid Optimizers Applicable to Large-Scale Design Problem

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***Abstract*—Design-Informatics has three points of view. One of these points is the investigation of efficient optimization to generate hypothetical database for a large-scale design problem. The results of the present study indicates the hybrid method between differential evolution and genetic algorithm is better performance for efficient exploration in design space under the condition for large-scale engineering design problem within 10^2 order evolution at most.**

***Keywords*-design-informatics; evolutionary computation; hybrid optimizer; differential evolution; genetic algorithm; particle swarm optimization;**

I. Introduction

The results produced by multi-objective (MO) optimization are not an individual optimal solution but rather an entire set of optimal solutions. This set generated by an MO optimization can be considered a hypothetical design database. Then, data mining techniques can be applied to this hypothetical database to acquire not only useful design knowledge but also structuring and visualizing of design space. This approach was suggested as design-informatics[1]. The design problem is firstly defined as objective functions, constraints, and design space. And then, optimization is implemented to obtain non-dominated solutions for database construction. The purpose of this approach is the conception support for designers in order to materialize innovation. This methodology is constructed by the three essences as problem definition, optimization, and information mining. In this study, optimizer for efficient exploration in design space is focused because the quality of hypothetical design database depends on that. The objective of this study is the evaluation of several evolutionary algorithms and their hybrid methods.

In this study, the practical engineering application with large evaluation time is assumed. Therefore, the evolutionary optimizer which efficiently explores in a small number of generations is needed. Differential evolution has recently better performance than genetic algorithm in MO optimization[2]. Then, the performance of genetic algorithm (GA)[3], differential evolution (DE)[4], particle swarm optimization (PSO)[5](GA and DE have advantage for global search while PSO is advantage for local search), and their hybrid methods is validated to employ practical engineering applications. Hence, they evaluate under the condition of a small number of population and generation. The qualitative performance is evaluated for the mathematical test functions for which the influence of noise are considered.

II. Optimizers

A. Hybrid Algorithm

Three optimizers, as GA, DE, and PSO, are coupled. First, multiple solutions are generated randomly as an initial population. Then, objective function values are evaluated for each solution. After the evaluation, the populations is equally divided into sub-populations for the operations in each optimizer(as this sub-population size can be decided in every generation, pure GA can be single performed when the sub-populations of DE and PSO are zero for example). New solutions generated by each operation are combined in the next generation. Non-dominated solutions in the combined population are archived. It is notable that only the archive data are shared among the each optimizer, the respective optimizers are independently carried out in the hybrid algorithm. Therefore, the total number of seven optimizers were evaluated as pure GA, pure DE, pure PSO, hybrid GA/PSO, hybrid DE/PSO, hybrid GA/DE, and hybrid GA/DE/PSO.

B. Configuration of GA Operators

Fonseca's Pareto ranking[6] and crowding distance[7] were used as the fitness value of each solution. The crowding distance was defined as the sum of Euclidean distances between the solution and its two nearest neighbors. As crossover operators, BLX-α[8] and UNDX[9] were used, which equally divided sub-population.

III. Problem Definition

A. Performance Metrics

Several performance measurement manners for evaluating the efficiency of MOEAs were suggested[10]. In this study, the following three metrics were used.

1) Convergence Metric: The first metric is *Convergence metric* Υ[11]. It measures the distance between the obtained non-dominated front Q and the set P^* of Pareto-optimum solutions as follows:

$$\Upsilon = \frac{1}{|Q|}\sum_{i \in Q} d_i, \qquad (1)$$

where d_i denotes the Euclidean distance in the objective-function space between the solution $i \in Q$ and the nearest member of P^*. The value near zero means better performance.

2) Cover Rate: The second metric is *Cover rate* R_c[12]. R_c evaluates the width and closeness of non-dominated solutions compared with Pareto-optimum front. The design space closed by the objective values from minimum to maximum is taken discretization. This metric describes the degree that non-dominated solutions cover discrete region. In this study, two-/three-dimensional test functions are evaluated. The objective-function space is separated by squares and cubes. The cover rate R_c is the following equation:

$$R_c = \frac{N_{\mathrm{NDS}}}{N_{\mathrm{Pareto}}}, \tag{2}$$

where N_{NDS} denotes the number of the cubes included in the derived non-dominated solutions. N_{Pareto} denotes the number of the cubes intersected by the Pareto front. The maximum value of R_c gives one and the minimum value of R_c gives zero, and then the value near one means better performance.

3) Hypervolume: The hypervolume indicator (or S metric) is described as the Lebesgue measure Λ of the union of hypercubes a_i defined by a non-dominated point m_i and a reference point x_{ref}[13]:

$$\begin{aligned} S(M) &\stackrel{\mathrm{def}}{\equiv} \Lambda\left(\left\{\bigcup_i a_i | m_i \in M\right\}\right) \\ &= \Lambda\left(\bigcup_{m \in M} \{x | m \prec x \prec x_{\mathrm{ref}}\}\right) \end{aligned} \tag{3}$$

B. Test Function

Three standard test function problems are employed in order to evaluate the performance of optimizers under the consideration of noise described by using normal distribution with random number. The first function is DTLZ3[14] without noise using three objective functions and 10 design variables. The second function is ZDT1[15] with/without noise, which is a simple two-dimensional problem with 10 design variables. The final function is TNK[16] with noise as a constraint two-dimensional test function. It is notable that noise is described by using normal distribution with random number. When an optimizer is applied to practical problems, experimental and computational values are employed as those of objective functions. Experiment includes error due to the flow quality in wind tunnel. Computation (computational fluid dynamics *etc.*) similarly has error due to mesh and various modeling *etc*. That is, as noise is occurred for the evaluated value under an identical condition, the consideration of noise is important to investigate the performance of optimizer applicable to practical engineering problem.

1) DTLZ3: This is a generic sphere problem. The Pareto-optimal surface always occurs for the minimum of $g(\boldsymbol{x})$ function. The number of design variables and objective functions set in this paper were 10 and three for DTLZ3.

$$\begin{aligned} &\text{Minimize:} && f_1(\boldsymbol{x}) = \cos\left(\frac{\pi}{2}x_1\right)\cos\left(\frac{\pi}{2}x_2\right)(1+g(\boldsymbol{x})) \\ &\text{Minimize:} && f_2(\boldsymbol{x}) = \cos\left(\frac{\pi}{2}x_1\right)\sin\left(\frac{\pi}{2}x_2\right)(1+g(\boldsymbol{x})) \\ &\text{Minimize:} && f_3(\boldsymbol{x}) = \sin\left(\frac{\pi}{2}x_2\right)(1+g(\boldsymbol{x})) \\ &\text{subject to:} && g(\boldsymbol{x}) = 100\times \\ &\left[k + \sum_{k=3}^{K}\left\{(x_k-0.5)^2 - \cos\left(20\pi\left(x_k-0.5\right)\right)\right\}\right] \geq 0, \\ & && 0 \leq x_k \leq 1, \quad k = 1,2,\cdots,K, \quad K = 10. \end{aligned} \tag{4}$$

The Pareto-optimum solution corresponds to $x_i = 0.5$ (for all $x_i \in \boldsymbol{x}$) and the objective function values lie inside the first octant of the unit sphere $\sum_{m=1}^{3} f_m = 1$ in a three-objective plot. All local Pareto-optimal fronts are parallel to the global Pareto-optimal front and an MOEA can get stuck at any of these local Pareto-optimal fronts, before converging to the global Pareto-optimal front.

2) ZDT1: As a test problem with noise, the following two-dimensional test function was considered:

$$\begin{aligned} &\text{Minimize:} && f_1(\boldsymbol{x}) = x_1 \\ &\text{Minimize:} && f_2(\boldsymbol{x}) = g(\boldsymbol{x})\left(1 - \sqrt{\frac{f_1(\boldsymbol{x})}{g(\boldsymbol{x})}}\right) \\ &\text{subject to:} && g(\boldsymbol{x}) = 1 + 9\cdot\frac{1}{K-1}\sum_{k=2}^{K} x_k, \\ & && 0 \leq x_k \leq 1, \quad k = 1,2,\cdots,K, \quad K = 30. \end{aligned} \tag{5}$$

The Pareto-optimum front is formed with $g(\boldsymbol{x}) = 1$. As noise is appended to this test function, the performance for noise occurred in practical problems is confirmed.

3) TNK: As a test problem with noise, the following two-dimensional test function was considered:

$$\begin{aligned} &\text{Minimize:} && f_1(\boldsymbol{x}) = x_1 \\ &\text{Minimize:} && f_2(\boldsymbol{x}) = x_2 \\ &\text{subject to:} && c_1(\boldsymbol{x}) = x_1^2 + x_2^2 \\ & && \quad -1 - 0.1\cos\left(16\arctan\frac{x_2}{x_1}\right) \geq 0 \\ & && c_2(\boldsymbol{x}) = \left(x_1 - \frac{1}{2}\right)^2 + \left(x_2 - \frac{1}{2}\right)^2 \leq \frac{1}{2} \\ & && 0 < x_i \leq \pi, \quad i = 1,2. \end{aligned} \tag{6}$$

This is a two real-valued variable constrained test problem. Since the function is simple and the objective-function space corresponds to the design-variable space, the Pareto front is determined by the constraints. As this function is a minimization problem, the discontinuous region which is not dominated by the other region in the curve described by $c_1(\boldsymbol{x}) = 0$. The ratio which the feasible region accounts

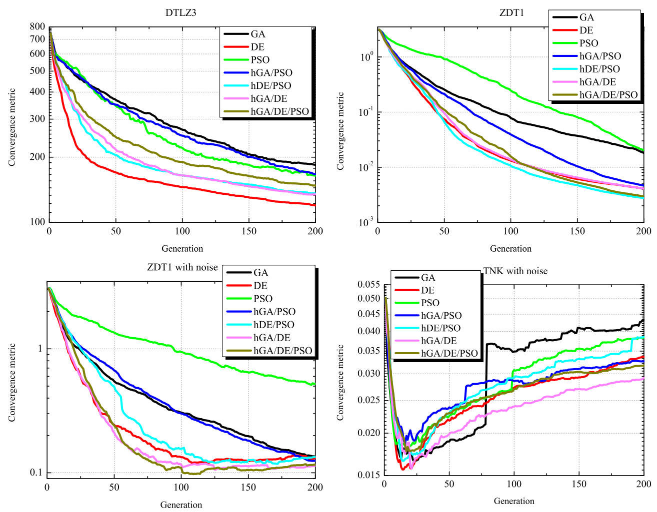

Figure 1. Histories of convergence metric for each test function.

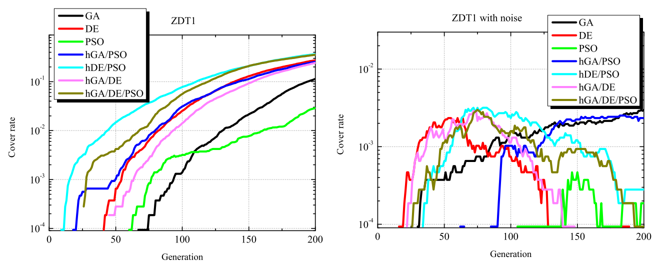

Figure 2. History of cover rate for each test function. As all data for DTLZ3 and TNK are zero, their histories were omitted.

is approximately 5% of the whole region. The Pareto front of this test function is non-convex surface. Therefore, this test function with noise reveals the performance for intricate practical problems.

IV. Results

The population size and the maximum number of generations were respectively set on 18 and 200. As the purpose of performance evaluation for several optimizations is to be applied to large-scale and real-world engineering design problem(for example, it takes one week for one-generation evaluation), comparatively small values were used. It is notable that the average values of 20 runs with different initial populations generated randomly were employed for evaluation.

The histories of convergence metric shown in Fig. 1 reveal that pure DE and the hybrid methods including DE have good performance. DE sustains damageless from noise. GA does not have much influence from noise. Although pure PSO has poor performance regarding noise, the hybridization with DE improve it. The hybridization between GA and DE gives the potentiating effect for the performance. Although the hybridization between PSO and the others also gives

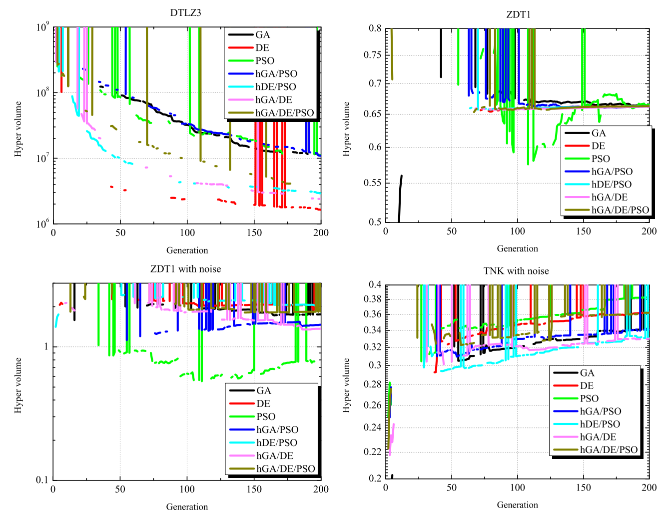

Figure 3. History of hypervolume for each test function.

similar effects, the frailty of pure PSO for noise is bottleneck.

The histories of cover rate shown in Fig. 2 also reveal that pure DE and the hybrid methods including DE have robustness for noise. GA does not have good performance. DE has adamant performance for noise, and also the hybrid methods including DE maintain similar robustness. Although pure PSO is frail for noise, the hybridization including PSO has compatibility.

The histories of hypervolume indicator shown in Fig. 3 also reveal that pure DE does not have good performance in the case without noise, but has robustness in the case with noise. DE has stable performance. Pure GA has good performance in the case without noise, but it is not good and problem dependency in the case with noise. Pure PSO is unstable in spite of noise and its performance depends on the test problems. In the case without noise, the hybridization between GA and PSO because DE drag away the bad performance. However, the hybridization including DE is good for the problem with noise due to the restoration of pure DE performance. Note that there is no meaningful difference regarding the results for TNK.

As a result, a hybrid method between GA and DE will be selected to apply to a large-scale engineering design problem because pure DE is robust and stable for noise and pure GA is expected to have latent performance due to complex operator compared with DE. PSO which does not have robustness regarding noise and the hybrid method with PSO should not be selected because practical engineering design problem certainly includes noise.

V. Conclusion

Pure and hybrid optimizers among genetic algorithm, differential evolution, and particle swarm optimization have been evaluated under the condition for large-scale and real-world engineering design problem. Three performance metrics were employed for three standard test function problems under the consideration of noise. Consequently, differential evolution and the hybrid optimizer including it have robustness for noise. The hybrid method between differential evolution and genetic algorithm should be used and improved for large-scale practical engineering design problem.

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